High Q² Probe of Nuclear Spectral Function and Color Transparency

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Abstract

Contrary to widespread opinion, color transparency (CT) brings essential ambiguity to, rather than helps in, study of the nuclear spectral function in quasielastic lepton scattering, $A(l, l'p)A'$, at high Q^2. Although the nuclear attenuation vanishes, the final state interaction (FSI) of a small-size ejectile wave packet, propagating through nuclear matter, remains. It manifests itself in a substantial, but uncertain, longitudinal momentum transfer to the nuclear medium. We predict a strong Fermi-momentum bias of the nuclear transparency at high Q^2, which enters as a factor at the nuclear spectral function, and makes uncertain the results of measuring the high-momentum tail of Fermi distribution.

According to expectations of PQCD the nuclear attenuation in quasielastic electron scattering, $A(e, e'p)A'$, vanishes at high Q^2, since the ejectile average size is small and it is supposed to demonstrate color transparency (CT) effects[1,2]. So one may conclude naively that the ejectile proton carries undisturbed information about the initial Fermi momentum of the struck proton due to suppression of FSI by CT. Thus one gets a perfect tool for study of the nuclear spectral function.

The purpose of this talk is to demonstrate that CT, on the contrary to above expectations, does not rule out the problem of FSI. Moreover, it makes it much more difficult than at low Q^2. Just CT breaks factorization of the hard quasielastic scattering cross section, is responsible for an longitudinal momentum transfer to the nucleus during FSI, and causes an asymmetry of nuclear transparency on missing momentum[3]. These facts leave no hope to use a high-Q^2 quasielastic scattering of leptons and hadrons for a reliable measurement of the nuclear spectral function.

Let us consider the imaginary part of the amplitude corresponding to all intermediate particles being on mass shell. To have CT different intermediate states have to cancel each other in the imaginary part of the amplitude. Elastic electron scattering on a free proton target corresponds to a fixed Bjorken variable, $x_B = 1$, which is fixed only by electron momenta. This fixes the mass of the ejectile at proton mass. We are arriving at a puzzling conclusion that only proton, rather than a set of different hadronic states, is produced. Thus no cancellation is possible, i.e. no CT.

However in quantum mechanics one has to define how the size of the ejectile is measured, because the measurement it self is known to affect the result. In order to minimize such an influence, one could put a size detector (second scattering center) far apart from the target proton. Then the above conclusion is correct: due to the time evolution of the ejectile only a proton will reach the distant detector.

To observe CT one has to put the size detector at a short distance from the target proton. Then the proton can not be treated as at rest, according to the uncertainty principle. Its momentum is distributed with a width of the order of the inverse distance from the detector (Fermi motion). As a result, some heavier states can be produced. The closer the target is to the size detector the more states are produced, the complete is the cancellation.

Thus the Fermi motion of a bound nucleon is a source of CT in quasielastic scattering[3,4]. On the other hand it restricts the amount of CT because the Fermi motion spectrum in nuclei is concentrated mainly within momenta of about $k_F = 0.2\text{GeV/c}$. If the Bjorken variable, $x_B = Q^2/(2m_p\nu)$, is fixed at $x_B = 1$, the electron knocks out a proton in the hard scattering on a bound proton, only if the latter was at rest. To produce an excited state of mass m^* in the hard scattering, the target nucleon must have an initial momentum in the direction opposite to the photon, $k_z \approx -(m^{*2} + m_p^2)/2\nu$. So the mass spectrum of produced states is restricted by:

$$m^{*2} < m_p^2 + 2\nu k_F \qquad (1)$$

Consequently even if the ejectile in $(e, e'p)$ reaction has a very small size, $\rho^2 \approx 1/Q^2$, the nucleus, as a quantum size detector, is insensitive to such a small size. Only the hadronic states, which can be created with available Fermi momenta contribute to the produced wave packet.Therefore a nucleus can resolve only size larger than:

$$\rho^2 \geq \frac{m_p}{k_F}\frac{1}{Q^2}. \qquad (2)$$

This restriction evidently reflects the fact that the size of the ejectile cannot be measured after the hard interaction at shorter distance than the mean internucleon separation in nuclei. The latter is just related to the mean Fermi momentum.

Let us consider the x_B-dependence of the typical observable,

$$Tr = \frac{\sigma^A}{A\,\sigma^N}, \qquad (3)$$

called *nuclear transparency* (or transmission coefficient), because it naively looks that when one assumes factorization of the nuclear cross section σ^A. Later on we will see that it is not so.

Starting with $x_B = 1$, note that the Fermi motion is used ineffectively: only a half of the quasielastic peak with $k_z < 0$ is used, no states on the mass shell are produced at Fermi-momenta $k_z > 0$. This situation is illustrated schematically in fig.1a. Varying x_B, one can shift the mass spectrum of hadronic states along the k_z axis, increasing or suppressing the interval of mass covered by the Fermi momenta. Thus one can effectively change the transverse size of the produced wave packet, and the handle to do it is the Bjorken variable[3].

For instance, if we decrease x_B down to $x_B \approx 1 - k_F/m_p$, the mass squared interval, Δm^2, doubles in comparison with $x_B = 1$. This is illustrated in fig.1b, and follows from the approximate relation between x_B, Fermi moment $\vec{k}$ and the produced mass:

$$M^2 \approx m_p^2 + Q^2 \, \frac{1 - x_B}{x_B} - 2qk_z, \tag{4}$$

where $\vec{q}$ if the 3-momentum of the photon. As a consequence of enlarged available mass interval, Tr will rise. If one keeps decreasing x_B one can arrive even at nuclear antishadowing, $Tr > 1$. Indeed at some x_B the lighest hadronic state, proton, is pushed out of the Fermi distribution, i.e. its direct production is extremely suppressed. However it can be still effectively produced via some heavier intermediate state. The nuclear transparency (3), in the case of $(e, e'p)$ reaction should be normalized by the probability to have Fermi momentum k_z, corresponding to the proton production. Thus, at $x_B < 1 - k_F/m$ the Fermi-distribution suppresses the denominator of (3), than the numerator. Therefore Tr increases and may cross unity. The latter depends much on the form of the edge of the Fermi momentum distribution and the mass spectrum of produced states. An explicit example is given below.

One gets an opposite result increasing x_B. The larger is x_B, the less hadronic states contribute to the ejectile wave packet. At last at $x_B \geq 1 + k_F/m$ all states except a proton are pushed out of the Fermi distribution on the k_F scale, as is shown schematically in fig.1c. It means that in this case the ejectile is simply a proton, the Glauber approximation is exact (up to usual inelastic corrections, which are as small as in the total hA cross section).

Thus we predict an x_B-asymmetry of nuclear transparency[3], which is the direct reflection of the deep quantum mechanical origin of the CT phenomenon.

After the Fermi bias effect was claimed and estimated in Ref.3, the calculations were repeated by other authors[5,6]. Nucleon correlations in nuclear density matrix were taken into account more carefully in Ref.6. This provides a distortion of the effective Fermi momentum distribution by the absorption of the recoil proton in nuclear matter. As a result a weak x_B dependence of nuclear transparency proves to appear even in the Glauber approximation. The form of the edge of Fermi momentum distribution, and the high-momentum tail, are most important for the magnitude of nuclear transparency at $x_B \leq 1 - p_F/m_p$. Depending on it, the interplay between the direct production of proton and production via intermediate excitations, can result in a nuclear antishadowing, like in Ref.3, or a shadowing, like in Ref.6. Unfortunately the high-momentum tail of the Fermi distribution is poorly known. We disagree with the statement of Ref.6, that the lack of nuclear antishadowing, they got, is due to the coherency constraint, since the latter was completely included in the evolution operator of Ref.3.

Let us estimate roughly the scale of the effect. Nuclear transparency (3) is read,

$$Tr = \frac{\sum_\alpha \int d^2b \left| \int_{-\infty}^{\infty} dz \, e^{ik_z z} \psi_A^\alpha(b, z) \langle p | \hat{V}(z, \infty) | i \rangle \right|^2}{A \, W_A(k_z) \, |\langle p | i \rangle|^2}, \tag{5}$$

where $\vec{k}$ is the "missing momentum" in the $(e, e'p)$ reaction, defined as $\vec{k} = \vec{q} - \vec{p}_p$. Ψ_A^α is the nuclear wave function in the shell α. $W_A(k_z) = \int d^2k_T \, W_A(\vec{k})$, where $W_A(\vec{k})$ is

the distribution function of Fermi momenta in the nucleus. $\hat{V}(z, z')\, exp(ik_z(z - z'))$ is an operator of evolution of the ejectile wave packet in the nuclear medium,

$$i\frac{d}{dz}|P\rangle = \hat{V}|P\rangle, \tag{6}$$

$|P\rangle$ denotes the set of states $|p\rangle$, a proton, and $|p^*\rangle$, which is an effective state reproducing contribution of all the diffractive excitations of the proton. The center of gravity of these excited states is the mass, m^*, of $|p\rangle$. The evolution operator $\hat{V}$ has the form,

$$\hat{V} = \begin{pmatrix} p - i\sigma_{\text{tot}}^{hN}/2\,\rho_A(z) & \frac{\alpha}{\sqrt{1-\alpha^2}}i\sigma_{\text{tot}}^{hN}\rho_A(z) \\ \frac{\alpha}{\sqrt{1-\alpha^2}}i\sigma_{\text{tot}}^{hN}\rho_A(z) & p - q_{\text{hh}^*} - i\sigma_{\text{tot}}^{hN}/2\,\rho_A(z) \end{pmatrix} \tag{7}$$

where p is the proton momentum; q_{hh^*} is the longitudinal momentum transfer,

$$q_{\text{hh}^*} = \frac{m^{*2} - m^2}{2\,\nu} \tag{8}$$

The initial state $|i\rangle$ is assumed to be a noninteracting one due to CT. This puts constraints on its decomposition, $|i\rangle = \alpha\,|p\rangle + \sqrt{1 - \alpha^2}\,|p^*\rangle$. The parameter α is expressed in terms of the ratio of the forward diffractive dissociation and of the elastic scattering cross sections as:

$$\frac{\sigma_{\text{dd}}}{\sigma_{\text{el}}} = \frac{\alpha^2}{1 - \alpha^2} \tag{9}$$

Let us represent the nuclear density matrix in the form,

$$\rho(\vec{r}_1, \vec{r}_2) = \sum_\alpha \Psi_A^\alpha(\vec{r}_1)\Psi_A^{\alpha*}(\vec{r}_2), \tag{10}$$

in the form

$$\rho(\vec{r}_1, \vec{r}_2) = \rho_A(\vec{r})\,P_A(\vec{\Delta}), \tag{11}$$

where $\vec{r} = 1/2(\vec{r}_1 + \vec{r}_2), \vec{\Delta} = \vec{r}_2 - \vec{r}_1$.

The function $P_A(\vec{\Delta})$ satisfies the following conditions,

$$P_A(0) = 1 \tag{12}$$

$$\int d^2\Delta\, P_A(\vec{\Delta})\, e^{i\vec{k}\vec{\Delta}} = W_A(\vec{k}) \tag{13}$$

Using this, the expression (5 is transformed to the form,

$$Tr = \left[A\, W_A(k_z)\,|\langle p|i\rangle|^2\right]^{-1}\int d^2b \int_{-\infty}^{\infty} dz\,\rho_A(\vec{b}, z)$$

$$\int_{-\infty}^{\infty} d\Delta_z\, e^{ik_z\Delta_z}\, P_A(\Delta_z)\,\langle p|\hat{V}(z - \Delta_z/2, \infty)|i\rangle\,\langle p|\hat{V}(z + \Delta_z/2, \infty)|i\rangle^* \tag{14}$$

Using equation (6) we can write a recurrent relation,

$$\hat{V}(z \pm \Delta z/2, \infty) = \hat{V}(z \pm \Delta z/2, z)\hat{V}(z, \infty) \tag{15}$$

It follows from (13), that the distribution $P_A(\Delta_z)$ is narrow, $\langle \Delta_z^2 \rangle^{1/2} \approx \frac{\sqrt{6}}{k_F} \approx 2 \ fm$. So the function $\hat{V}(z \pm \Delta_z/2, z)$ may be expanded over this small parameter.

$$\hat{V}(z, z \pm \Delta_z/2) = \hat{I}(1 \pm ip\Delta_z/2) \mp \frac{i}{2} \, \Delta_z \, \hat{U} + O(\Delta_z^2), \tag{16}$$

where $\hat{I}$ is a unit matrix. We neglect the terms of the order of $\langle \Delta_z^2 \rangle$ and higher, using smallness of the correlation length. The conditions for such an approximation are:

$$\frac{1}{2} \, \Delta z \Delta p \ll 1 \tag{17}$$

$$\frac{1}{2}\sigma_{tot}\rho_A(\vec{r})\Delta z \ll 1 \tag{18}$$

The former is valid at high energies, starting from a few GeV. The latter depends on the nuclear density at the point of interaction. It is satisfied even at the very center of a nucleus, but becomes more exact at the nuclear edge, which is the most important for the process, if it is far from the saturation of CT.

Note that the terms of the order of $\langle \Delta_z^2 \rangle$ give rise to a weak k_z-dependence of Tr even in Glauber approximation (compare with Ref.6).

When (16) is substituted into (14), the imaginary part of $\hat{U}$ cancels. The real part of $\hat{U}$ gives an additional phase factor, $exp(i \, \Delta p\Delta z/2)$ to each amplitude $\langle p|\hat{V}(z, \infty)|p^* \rangle$. This shift of phase leads to the shift of the argument of $W_A(k)$ after integration over Δz. To take it into account let us introduce a shifted state

$$|\tilde{i}\rangle = \alpha|p\rangle + \sqrt{1 - \alpha^2} \, \sqrt{\frac{W_A(k + \Delta p)}{W_A(k)}} \, |p^*\rangle \tag{19}$$

Such a transformation of the initial state is an approximation (see Ref.6), which is numerically quite precise.

Finally, (14) is transformed into

$$Tr(x_B) = \frac{\int d^2b \, \int_{-\infty}^{\infty} dz \, \rho_A(b, z) \, |\langle p|\hat{V}(z, \infty)|\tilde{i}\rangle|^2}{A \, |\langle p|i\rangle|^2} \tag{20}$$

To evaluate we use the two-channel approximation[7]. We are not aimed here to provide reliable numerical predictions, but only raise principal questions, and need only to get a rough estimate of the effect. For this reason we use the simple two-channel approximation to evaluate $\hat{V}(z, \infty)|\tilde{i}\rangle$, and a simplified form of Fermi momentum distribution with a Gaussian parameterization,

$$W_A(k) = \frac{3}{2\pi k_F^2} \exp(-3k^2/2k_F^2). \tag{21}$$

The results are shown in fig.2 as a function of x_B for Q^2 equal to 7 GeV^2, 15 GeV^2 and 30 GeV^2. We use $m^* = 1.6 \ GeV$ and $\alpha = 0.1$. As mentioned previously, at $x_B > 1$ the nuclear transparency is small and close to the expectation of the Glauber model.

At $x_B < 1$, on the contrary excited states are preferentially produced; to the extent that the transparency even becomes larger then 1 for small x_B.

The above calculations have a demonstrative character. Nevertheless they are quite indicative of the results that would be obtained with a better calculation.

We see that the $(l, l'p)$ reaction at high Q^2 is a poor probe of the nuclear spectral function.

The same concerns apply to wide-angle quasi-elastic $(p, 2p)$ scattering (see next section). To conclude, a few comments in order.

- CT in quasielastic scattering on nuclei is possible only due to Fermi motion. However the finiteness of Fermi momenta strongly violates the CT sum rule. This depends on Bjorken x_B (or missing momentum), which allows to handle the amount of CT. The predicted x_B-asymmetry of nuclear transparency is a new observable of CT, reflecting its a deep quantum mechanical nature.

- The finiteness of available Fermi momenta and energy conservation cut off most of hadronic states, which contribute to the CT sum rule. One might arrive at a wrong conclusion, forgetting about it. An example is the proposal[21] for CEBAF to study CT effects in $(e, e'2p)$ on light nuclei. Even at low $Q^2 \approx 4 \ GeV^2$ the authors predict about 50% deviation from Glauber model. They use three channel model with masses $m = m_p$, $m^* = 1.6 \ GeV$ and $m^{**} = 3 \ GeV$. It is easy to check however, that two latter states are too heavy to be produced with reasonable Fermi momentum at $Q^2 = 4 \ GeV^2$. Thus this model, corrected for the conservation of energy, predicts no deviation from the Glauber model in the CEBAF range of Q^2.

- Although different hadronic states produced in the hard interaction use different Fermi momenta, the final proton momentum is fixed. The difference between the longitudinal momenta of hadronic states after the hard interaction, is compensated by longitudinal momentum transfer in diffractive FSI. Thus a small size ejectile, consisted of many hadronic waves, transfer to the nuclear matter a positive (relative to the photon direction) longitudinal momentum of the order of mean Fermi momentum. This keeps being valid at any Q^2. Therefore CT does not mean an overall suppression of FSI, but only strong cancellation in the transmission amplitude.

- According to the wide spread opinion, the high Q^2 quasielastic scattering, provides a clean tool for measurement of the nuclear spectral function, because CT suppresses FSI. Such a believe is based on the classical treatment of CT. It is amazing that a correct quantum mechanical analyses leads to the opposite conclusion: CT does not help, but just spoils any opportunity to measure the nuclear spectral function. Firstly, according to the previous comment, CT does mean that FSI exists, provided an additional longitudinal momentum transfer, escaping detection. It is unavoidable property of CT. Secondly, if one wants to measure the large momentum tail of Fermi

distribution he faces the Fermi bias of nuclear transparency, which cannot be calculated reliably. One is safe of these problems only at $x_B < 1 - p_F/m_p$, where the Glauber approximation is correct. One may wonder however in this case, why does he need high Q^2 at all.

- The observed phenomenon of Fermi bias sheds light[15] on the puzzling results of measurement of nuclear transparency in quasielastic $(p, 2p)$ scattering in the BNL experiment[7]. Actually the data were distributed over missing momentum, so the asymmetry predicted here for such a distribution has a direct concern to interpretation of the data. Calculations were performed in Ref.8 within the same two-component model, as used in Ref.3. It is found that the missing momentum asymmetry nicely explains the observed drop of transparency at 12 GeV/c beam momentum, which have just confused theorists, expecting a rising energy dependence.

- Note that it follow from the above consideration of quantum effects, that CT effects are essentially due to the observation of the ejectile proton in the final state. This is why no CT effect is expected in inclusive deeply inelastic scattering of leptons on nuclei.

- The finiteness of available Fermi momenta and energy conservation cut off most of hadronic states, which contribute to the CT sum rule. One might arrive at a wrong conclusion, forgetting about it. An example is the proposal[9] for CEBAF to study CT effects in $(e, e'2p)$ on light nuclei. Even at low $Q^2 \approx 4\ GeV^2$ the authors predict about 50% deviation from Glauber model. They use three channel model with masses $m = m_p$, $m^* = 1.6\ GeV$ and $m^{**} = 3\ GeV$. It is easy to check however, that two latter states are too heavy to be produced with reasonable Fermi momentum at $Q^2 = 4\ GeV^2$. Thus this model, corrected for the conservation of energy, predicts no deviation from the Glauber model in the CEBAF range of Q^2.

References

1. A.B. Zamolodchikov, B.Z. Kopeliovich and L.I. Lapidus, *JETP Lett.* **33**, (1981) 595
2. G. Bertsch, S.J. Brodsky, A.S. Goldhaber and J.G. Gunion, *Phys. Rev. Lett.* **47**, (1981) 297
3. B. Jennings ans B. Kopeliovich, *Phys. Rev. Lett.* **70**, (1993)3384
4. A. Biamconi, S. Boffi, D.E. Kharzeev, preprint FNT/T-92/38, (1992)
5. A. Biamconi, S. Boffi, D.E. Kharzeev, FTN/T-92/47,
6. N.N. Nikolaev et al., Jülich preprint KFA-IKP(Th)-1993-16(1992)
7. B.Z. Kopeliovich and L.I. Lapidus, *JETP Lett.* **32**, (1980)595
8. B. Jennings and B. Kopeliovich, PANIC XIII, Book of Abstracts, p.165
9. K. Egiyan et al., PANIC XIII, Book of Abstracts, p.178, 1993

Figure captions

Fig.1

Schematic picture, showing the distribution of exited proton states over the Fermi momentum, versus Feynman variable x_B.

Fig.2

The nuclear transparency in $(e, e'p)$ as function of Bjorken variable x_F, versus $Q^2 = 7,\ 15$ and $30\ GeV^2$